METHODS OF EVOLUTIONARY GAME THEORY

Summary

An overview of methods of mathematical modeling of evolutionary processes related to evolutionary game theory is presented. These methods are inspired by classical, economical game theory introduced by John von Neumann and Oskar Morgenstern and developed by others, including John Nash, one of the most important researchers involved in development of game theoretic methods. Evolutionary game theory was originated by John Maynard Smith in his pioneering papers with George Price and the book "Evolution and a theory of games". Now the theory is one of the most important tools for modeling evolution of behavior. Introducing replicator dynamics, a system of ordinary differential equations describing changes among frequencies, allowed evolutionary game theory to become an independent and self-consisted mathematical theory. There are discussed basic concepts underlying evolutionary game theoretic framework in the paper. At the beginning, elementary classical game theoretic notions (*strategy set*, *pure and mixed strategies*, *payoff function*) and *Nash equilibrium* concept are introduced. Then I focus on basic static conditions of evolutionary stability, such *evolutionarily stable strategy* (ESS), *evolutionarily stable state* (ESState) and *evolutionarily stable set* (ESSet). Then evolutionary game theoretic framework is extended to dynamical context by introducing replicator dynamics. Mathematical toolbox presented in previous sections is used to prepare an example of application of evolutionary games to biological problems. A rigorously derived and analyzed example is *hawk-dove game*, the model of selection between aggressive and peaceful individuals presented by John Maynard Smith and George Price in their pioneering paper. Also example of *rock-scissor-paper game* is briefly discussed. This is an important example from biological point of view, since it describes cycling dynamics among different male mating strategies of *Uta stansburiana* lizards. Then extensions of basic evolutionary game framework and non-biological applications (in social sciences and economics) are discussed. The last section is devoted to famous *prisoners dilemma*. This is a canonical model used in modeling of evolution of cooperation.